

Math 55 Discussion problems 19 & 24 Jan

1. Let p and q be the propositions

p : I bought a lottery ticket this week.

q : I won the million dollar jackpot.

Express each of these propositions as an English sentence.

(a) $\neg p$

(c) $p \rightarrow q$

(e) $p \leftrightarrow p$

(g) $\neg p \wedge \neg q$

(b) $p \vee q$

(d) $p \wedge q$

(f) $\neg p \rightarrow \neg q$

(h) $\neg p \vee (p \wedge q)$

2. Let p , q , and r be the propositions

p : You get an A on the final exam.

r : You get an A in this class.

q : You do every exercise in this book.

Write these propositions using p , q , and r and logical connectives (including negations).

(a) You get an A in this class, but you do not do every exercise in this book.

(b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

(c) To get an A in this class, it is necessary for you to get an A on the final.

(d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

(e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

(f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

3. Use a truth table to verify the first De Morgan law $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

4. Translate these statements into English, where $R(x)$ is “ x is a rabbit” and $H(x)$ is “ x hops” and the domain consists of all animals.

(a) $\forall x(R(x) \rightarrow H(x))$

(c) $\exists x(R(x) \rightarrow H(x))$

(b) $\forall x(R(x) \wedge H(x))$

(d) $\exists x(R(x) \wedge H(x))$

5. Let $Q(x, y)$ be the statement “Student x has been a contestant on quiz show y .” Express each of these sentences in terms of $Q(x, y)$, quantifiers, and logical connectives, where the domain for x consists of all students at your school and for y consists of all quiz shows on television.

- (a) There is a student at your school who has been a contestant on a television quiz show.
 - (b) No student at your school has ever been a contestant on a television quiz show.
 - (c) There is a student at your school who has been a contestant on Jeopardy! and on Wheel of Fortune.
 - (d) Every television quiz show has had a student from your school as a contestant.
 - (e) At least two students from your school have been contestants on Jeopardy!.
6. For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.
- (a) “If I play hockey, then I am sore the next day.” “I use the whirlpool if I am sore.” “I did not use the whirlpool.”
 - (b) “If I work, it is either sunny or partly sunny.” “I worked last Monday or I worked last Friday.” “It was not sunny on Tuesday.” “It was not partly sunny on Friday.”
 - (c) “All insects have six legs.” “Dragonflies are insects.” “Spiders do not have six legs.” “Spiders eat dragonflies.”
 - (d) “Every student has an Internet account.” “Homer does not have an Internet account.” “Maggie has an Internet account.”
 - (e) “All foods that are healthy to eat do not taste good.” “Tofu is healthy to eat.” “You only eat what tastes good.” “You do not eat tofu.” “Cheeseburgers are not healthy to eat.”
 - (f) “I am either dreaming or hallucinating.” “I am not dreaming.” “If I am hallucinating, I see elephants running down the road.”